The First Notebook

First prep

Geometry

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Geometric Concepts

1 The line segment:

It is a set of points consisting of two distinct points and all points between them when we join them by a ruler.

- denoted by $\overline{AB}$ or $\overline{BA}$

- The line segment has two ends and a length.
- The length of $\overline{AB}$ is denoted by AB or BA

2 The ray:

It is a line segment extended from only one of its terminals without limit.

- denoted by $\overrightarrow{AB}$

- $\overrightarrow{AB}$ differs $\overrightarrow{BA}$
- The ray has a starting point and it has no end point, therefore it has no length.

3 The straight line:

If we extend the line segment in both directions infinitely, we will get a straight line.

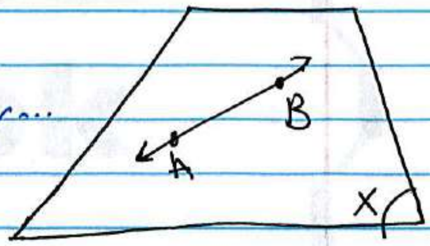
- denoted by $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$

- The straight line doesn't have a starting point or an end point, hence the straight line has no length.

Remark. $\overline{AB} \subset \overrightarrow{AB} \subset \overleftrightarrow{AB}$

4 The plane:

It is a flat and unbounded surface, and it is extended without Limit in all directions.



- It is denoted by X , Y, Z or...
- $\overleftrightarrow{AB} \subset X$

5 The angle:

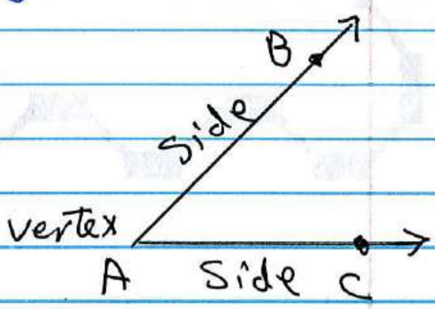
It is the union of two rays with the same starting point.

i.e.

$\overrightarrow{AB} \cup \overrightarrow{AC} = \angle CAB$

or $\angle BAC$

or $\angle A$



6 Measurement of The angle:

- It is the number expressing the amount of happened divergence between the two sides.
- The angle is measured using degree unit which is denoted by (°)

where

$1^\circ = 60'$, $1' = 60''$

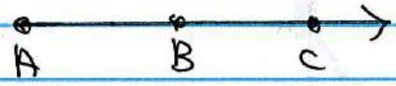
1° degree , $1'$ minute , $1''$ second

The types of angles:

The angles are classified according to their measures as follows:

1 Zero angle

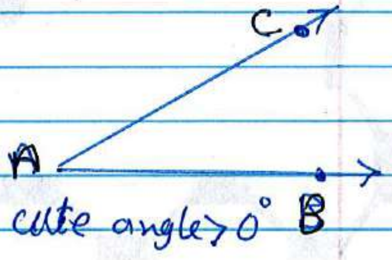
Its measure = 0°
its sides are coincident



$m(\angle BAC) = 0^\circ$

2 Acute angle:

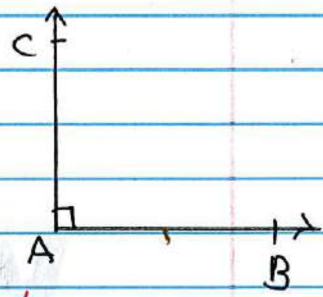
Its measure more than 0° and less than 90°
i.e $90^\circ >$ measure of acute angle $> 0^\circ$



3 Right angle:

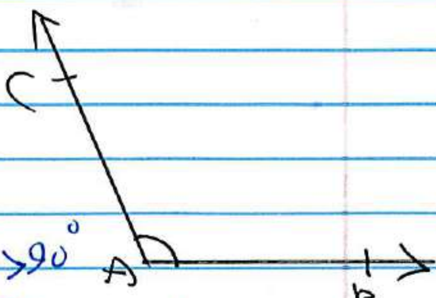
Its measure = 90°

$m(\angle BAC) = 90^\circ$

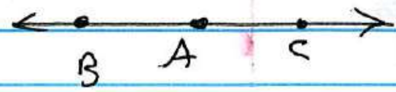


4 obtuse angle:

Its measure is more than 90° and less than 180°.
i.e
$180^\circ >$ measure of obtuse ang $> 90^\circ$



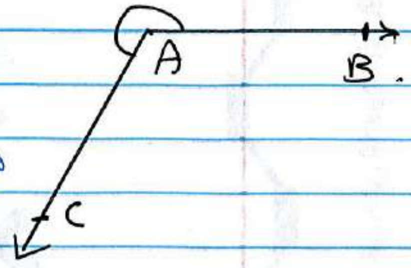
5 Straight angle:



its measure = 180
i.e. $m(\angle BAC) = 180^\circ$

6 Reflex angle:

Its measure is more than 180°
and less than 360°
i.e $180° < m(\angle BAC) < 360°$



Remark: In the opposite figure:

$m(\angle BAC) + m(\text{reflex} \angle BAC) = 360$

i.e

$m(\text{reflex} \angle BAC) = 360 - m(\angle BAC)$

Ex: Mention the type of each of the angles which their measures are as follows:

1. 45° ⇒ It's an acute angle
2. 96° ⇒ It is an obtuse angle
3. 90° ⇒ It is a right angle
4. 160° ⇒ It's an obtuse angle
5. 240° ⇒ It is a reflex angle
6. 125° 30′ ⇒ It is an obtuse angle
7. 180° ⇒ It is a straight angle
8. 89° 60′ ⇒ 89° 60′ = 90° It's a right angle
9. 179° 59′ 60″ ⇒ 179° 59′ 60″ = 180° It's a straight
10. 89° 62′ ⇒ 89° 62′ = 90° 2′ It is an obtuse

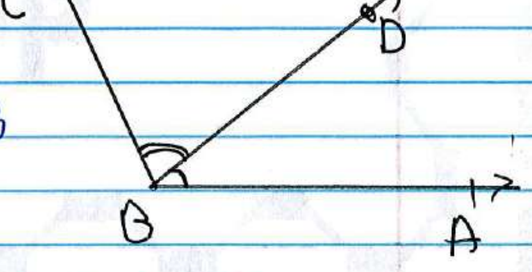
Some relations between the angles

Adjacent angles:

two angles are said to be adjacent if they have a common vertex and a common side and The other sides are on opposite sides of this common side.

In the opposite figure:

- $\angle ABD$ and $\angle DBC$ are two adjacent angles
- The have a common vertex B and common side $\overrightarrow{BD}$



Complementary angles:

two angles are said to be complementary if The sum of their measures is 90°

Remarks:

- the complement of acute angle is acute angle
- the complement of zero angle is right angle
- The complement of the same angle (or equal angles in measure) are equal in measure.

i.e.

if $\angle A$ complements $\angle B$, $\angle C$ complements $\angle B$, then $m(\angle A) = m(\angle C)$

Supplementary angles:

Two angles are said to be supplementary if the sum of their measures is 180°

Remarks:

- The supplement of an obtuse angle is an acute angle
- the supplement of a right angle is a right angle
- the supplement of a zero angle is a straight angle
- the supplements of the same angle (or equal angles in measure) are equal in measure.

i.e. If $\angle A$ supplements $\angle B$ and $\angle C$ supplements $\angle B$, the $m(\angle A) = m(\angle C)$.

Ex: Complete the following table:

measure of the angle	30	90	40°	acute
measure of its complement	60°	0°	50°	acute

- the complement of an angle $= 90^\circ -$ the measure of this angle

Ex: Complete the following table:

measure of the angle	30°	110°	0°	90°	acute
measure of its supplement	150°	70°	180°	90°	obtuse

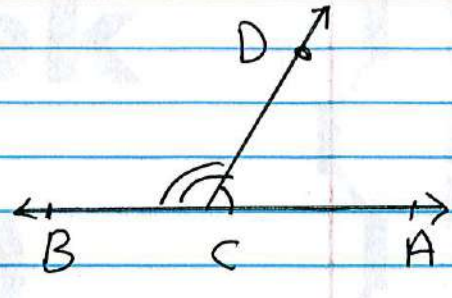
The supplement of an angle = ...

$= 180^\circ -$ the measure of this angle

* The two adjacent supplemetary angles:-

Two adjacent angles formed by a straight line and aray with astarting point on this straight line, are supplementary.

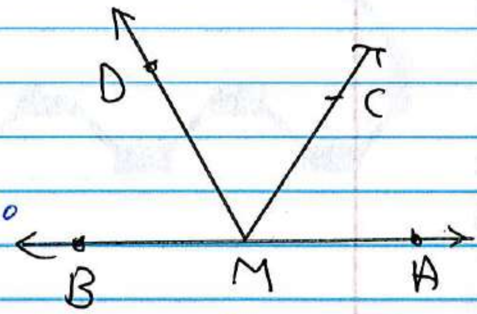
In the opposite figure:



If $\overleftrightarrow{AB} \cap \overrightarrow{CD} = \{C\}$, Therefore,

$m(\angle ACD) + m(\angle DCB) = 180^\circ$
"Sraight angle"

Remark:



- If $M \in \overleftrightarrow{AB}$ Then

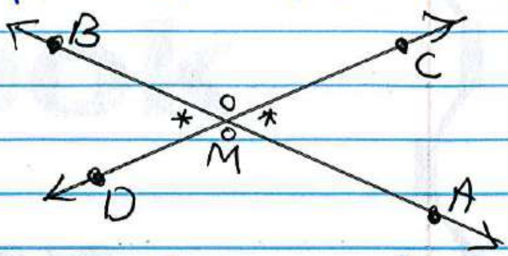
$m(\angle AMC) + m(\angle CMD) + m(\angle DMB) = 180^\circ$

- the outer sides of the two adjacent supplementary angles are on the same straight line
- The outer sides of the two adjacent complementary angles are perpendicular.
- If two adjacent angles are not supplementary, Then the outer sides are not on the same straight line

Vertically opposite angles (V.O.A):

If two straight lines intersect, then the measures of each two vertically opposite angles are equal.

In the opposite figure:



If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$ then,

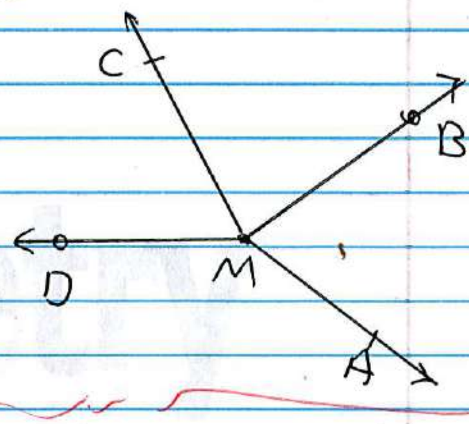
$m(\angle AMC) = m(\angle BMD)$ (V.O.A)

also, $m(\angle CMB) = m(\angle AMD)$ (V.O.A)

Accumulative angles at a point:

The sum of the measures of the accummulative angles at a point is $360°$

In the opposite figure:

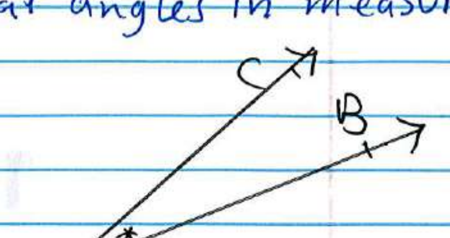


$m(\angle AMB) + m(\angle BMC) + m(\angle CMD)$
$+ m(\angle DMA) = 360°$

The angle bisector:

It is the ray that devides the angle into two halves (two equal angles in measure)

In the opposite figure:



$\overrightarrow{MB}$ bisects $\angle AMC$

i.e

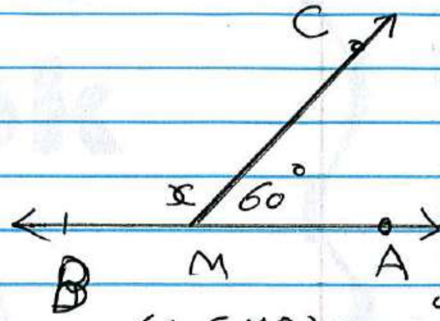
$m(\angle AMB) = m(\angle BMC) = \frac{1}{2} m(\angle AMC)$

In each of the following figures, find the measure of the required angles under each figure:

Since $M \in \overleftrightarrow{AB}$

then: $m(\angle CMB) = 180 - 60 = 120°$

Because $m(\angle AMB) = 180$ "straight angle"

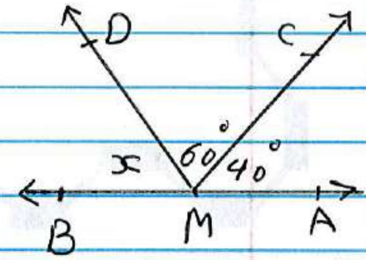


$m(\angle CMB) = \dots°$

Since $M \in \overleftrightarrow{AB}$

Then: $m(\angle DMB) = 180 - (60 + 40)$
$= 180 - 100$
$= 80°$

Because $\angle AMB$ is a straight angle

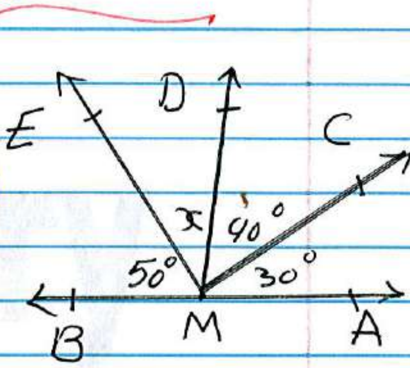


$m(\angle DMB) = \dots°$

Since $M \in \overleftrightarrow{AB}$

then: $m(\angle DME) = 180 - (50 + 30 + 40)$
$= 180 - 120$
$= 60°$

Because: $m(\angle AMB) = 180°$ "straight angle"

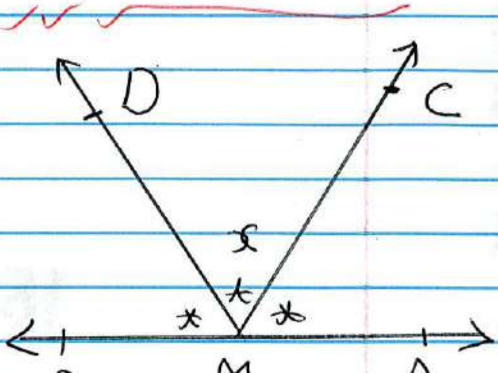


$m(\angle DME) = \dots$

Since $M \in \overleftrightarrow{AB}$

therefore: $m(\angle AMD) = 180$

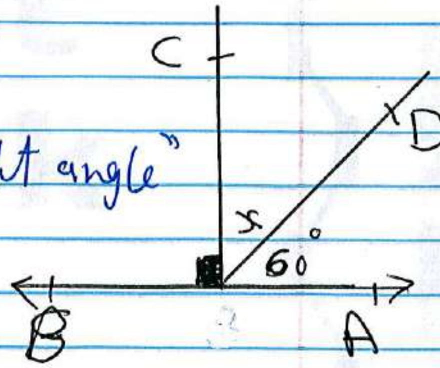
then: $m(\angle CMD) = \frac{180}{3} = 60°$



$m(\angle CMD) = \dots$

In of the following find the value of x

if $\overline{MC} \perp \overleftrightarrow{AB}$



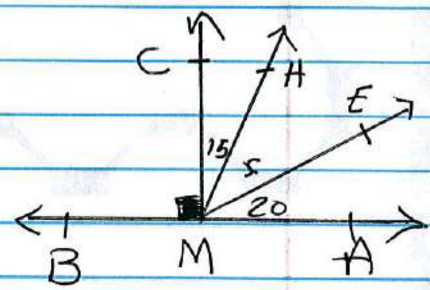
Solution: $M \in \overleftrightarrow{AB}$

then: $m(\angle AMB) = 180°$ "straight angle"

Since $\overline{MC} \perp \overleftrightarrow{AB}$

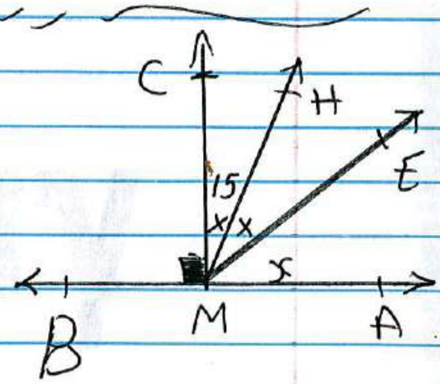
therefore, $m(\angle CMB) = 90°$

$x = 180 - (60 + 90) = 180 - 150 = 30°$



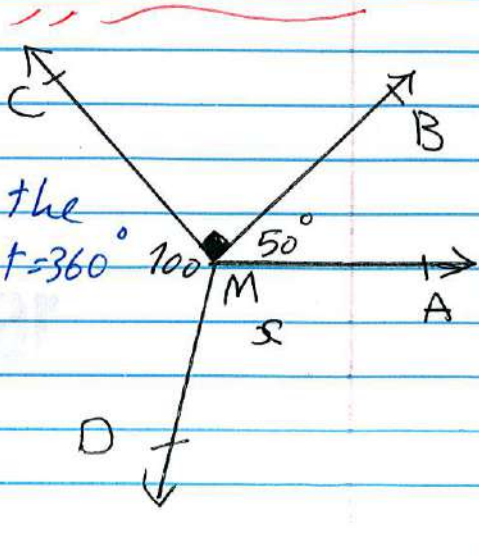
$x = 180 - (90 + 15 + 20)$

$= 180 - 125 = 55°$



$x = 180 - (15 + 15 + 90)$

$= 180 - 120 = 60°$

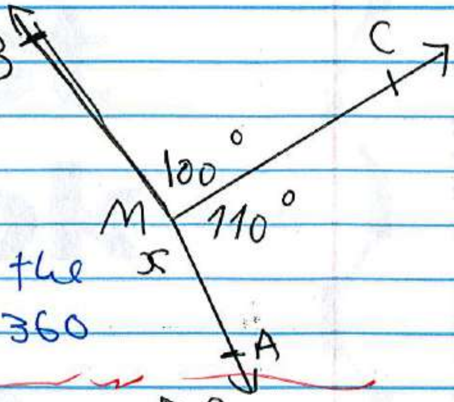


The sum of the measures of the accumulative angles at a point $= 360°$

$x = 360 - (50 + 90 + 100)$

$= 360 - 240 = 120°$

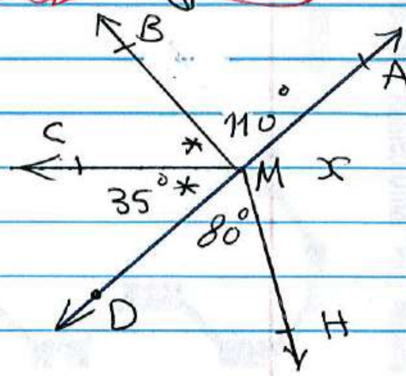
In each of the following figures find the value of x.



Solution

$x = 360 - (110 + 100) = 360 - 210$
$= 150°$

Because the sum of measures of the accumulative angles at a point $= 360$



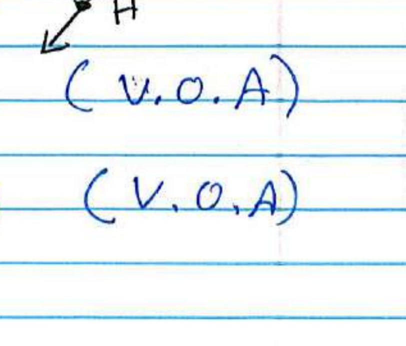
$x = 360 - (110 + 35 + 35 + 80)$

$= 360 - 260 = 100°$

Because the sum of measures of the accumulative angles at a point $= 360°$

In the opposite figure:-

$\overleftrightarrow{AD} \cap \overleftrightarrow{BH} = \{M\}$,
$m(\angle CMD) = 28°$ and $m(\angle BMC) = 115°$
find the measures of the angles marked by (?)



Solution

- $m(\angle AMB) = 180° - (115° + 28°)$
  $= 180 - 143 = 37°$

Because $m(\angle AMD) = 180°$ "straight angle"

- $m(\angle DMH) = m(\angle BMA) = 37°$ (V.O.A)

- $m(\angle AMH) = m(\angle BMD) = 143$ (V.O.A)

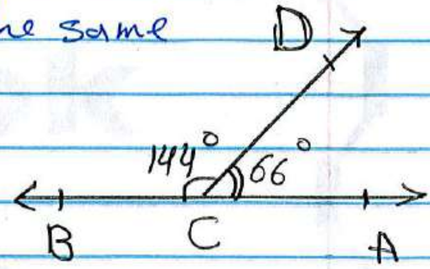
In each of the following figure, state if $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the straight line or not? and why?

Solution:

$\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the the same straight line

because:

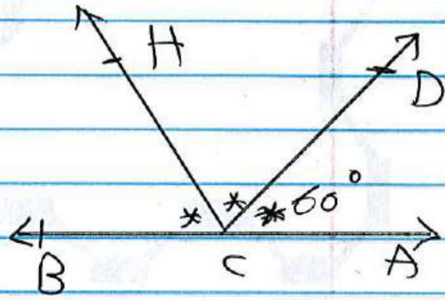
$m(\angle ACD) + m(\angle DCB)$
$= 144 + 66 = 180°$



$\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line.

Because:

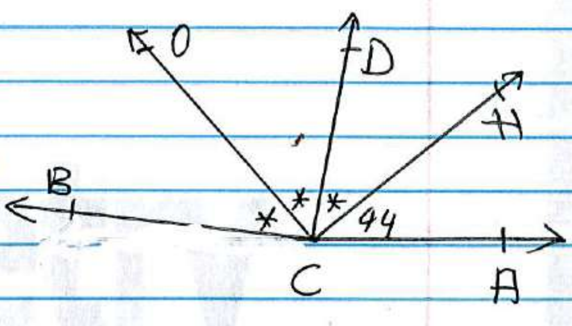
$m(\angle ACB) = 3 \times 60 = 180°$



$\overrightarrow{CA}$ and $\overrightarrow{CB}$ are not on the same straight line:

Because

$m(\angle ACB) = 4 \times 44 = 176 \neq 180°$

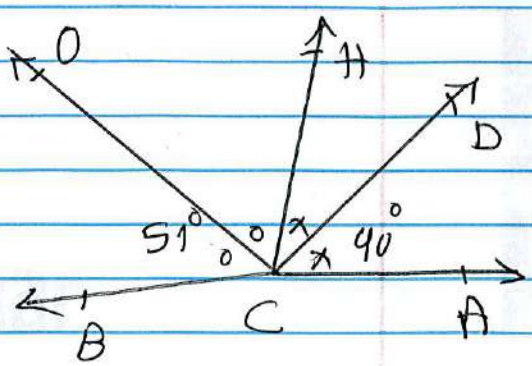


$\overrightarrow{CA}$ and $\overrightarrow{CB}$ are not on the same straight line:

Because:

$m(\angle ACB) = 40 + 40 + 51 + 51$

$= 182 \neq 180$



Congruence

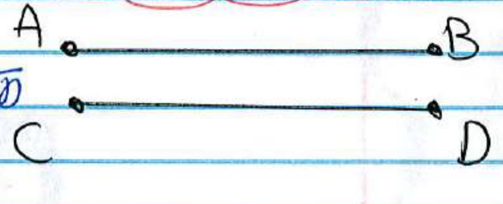
First: Congruence of two line segments:

Two line segments are congruent if they are equal in length.

i.e

If the length of $\overline{AB}$ = the length of $\overline{CD}$

then $\overline{AB}$ is congruent to $\overline{CD}$



and we write $\overline{AB} \equiv \overline{CD}$

And vice versa:

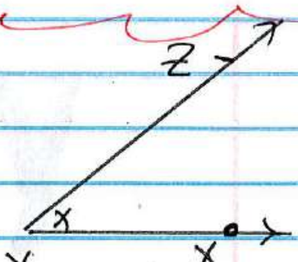
Every two congruent line segments are equal in length.

Second: Congruence of two angles:

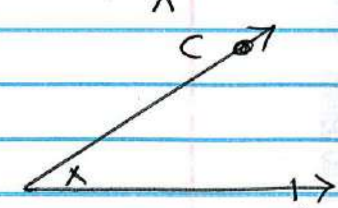
Two angles are congruent if they are equal in measure

i.e if $m(\angle ABC) = m(\angle XYZ)$

then $\angle ABC$ is congruent to $\angle XYZ$



and we write $\angle ABC \equiv \angle XYZ$



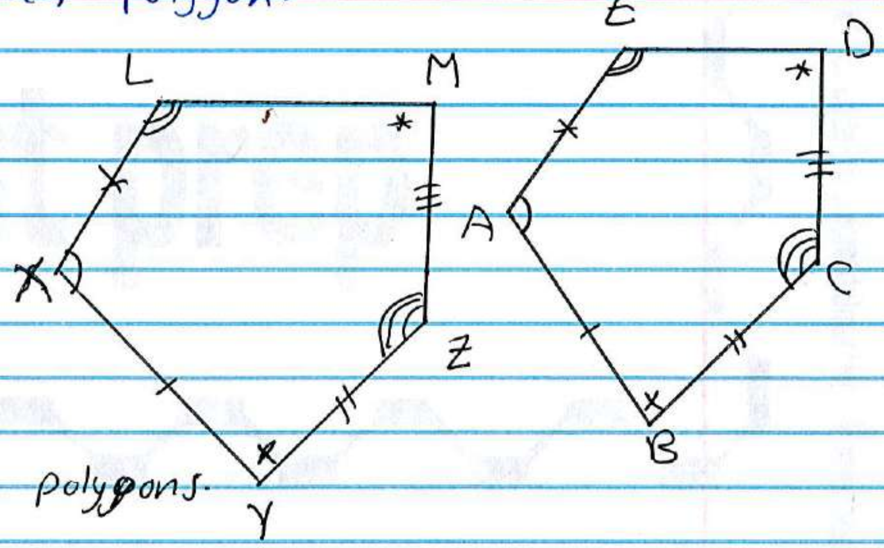
And vice versa:

Every two congruent angles are equal in measure.

Third: Congruence of two polygons:

Two polygons are congruent if there is correspondence between their vertices such that each side and each angle in the first polygon is congruent to its corresponding element in the other polygon.

for example:



In the opposite two polygons. if:

1. Each two corresponding sides are equal in length.

i.e. $AB = XY$, $BC = YZ$, $CD = ZM$, $DE = ML$ and $EA = LX$

2. Each two corresponding angles are equal in measure

i.e $m(\angle A) = m(\angle X)$, $m(\angle B) = m(\angle Y)$, $m(\angle C) = m(\angle Z)$
$m(\angle D) = m(\angle M)$ and $m(\angle E) = m(\angle L)$

then the polygon ABCDE is congruent to the polygon XYZML and we write

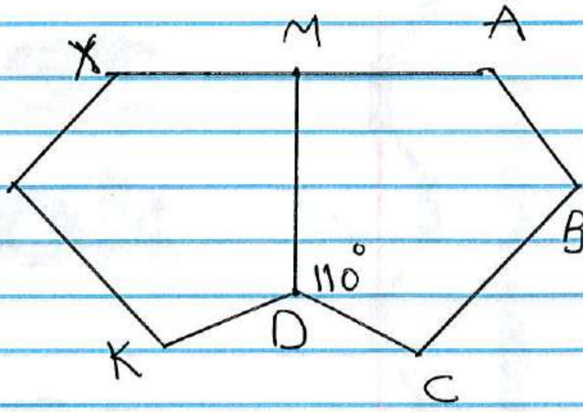
the polygon ABCDE $\equiv$ the polygon XYZML

In the opposite figure:

If the figure

ABCDM ≡ the figure XYKDM

Complete



① AB = ... , CD = ...

② BC = ... , XM = ...

③ $m(\angle A) = m(\angle ...)$, $m(\angle B) = m(\angle ...)$

④ $m(\angle C) = m(\angle ...)$, $m(\angle CDM) = m(\angle ...)$

⑤ the axis of symmetry of figure ABCDKYX is ...

⑥ $m(\angle AMD) = ...^\circ$

⑦ $m(\angle CDK) = ...^\circ$

⑧ $\overline{MD}$ is called.

Solution

we deduce from the Congruence of the two polygons

① AB = XY , CD = KD

② BC = YK , XM = AM

③ $m(\angle A) = m(\angle X)$, $m(\angle B) = m(\angle Y)$

④ $m(\angle C) = m(\angle K)$, $m(\angle CDM) = m(\angle KDM)$

⑤ The axis of symmetry is $\overline{MD}$

⑥ $m(\angle AMD) = \frac{180}{2} = 90^\circ$ So we deduce that $\overline{MD} \perp \overline{AX}$

⑦ $m(\angle CDK) = 360 - (110 + 110) = 360 - 220 = 140^\circ$

⑧ $\overline{MD}$ is called a Common Side

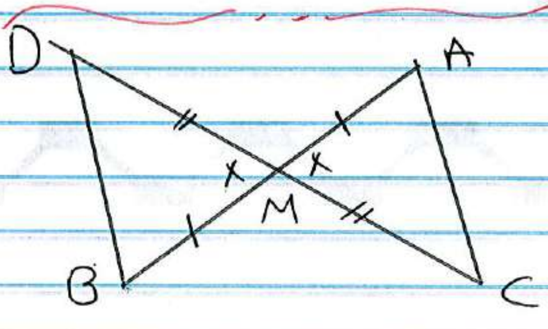
Congruent Triangles

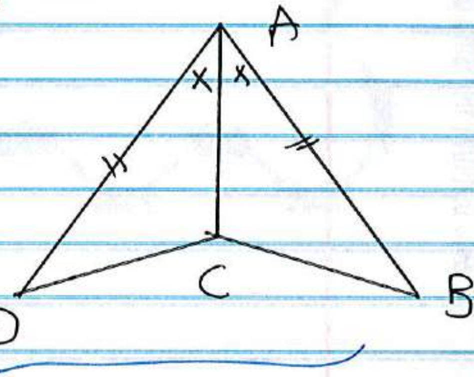
The cases of Congruence of two triangles:

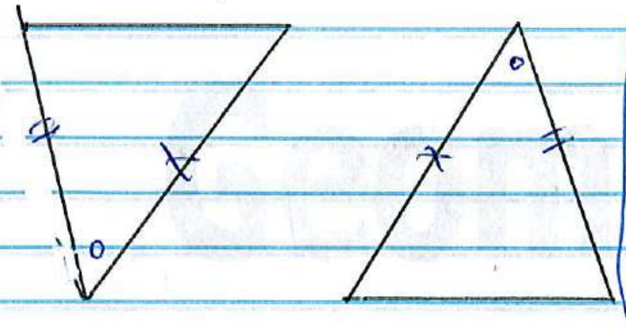
The first case of congruece of two triangles
(two sides and the included angle S.A.S)

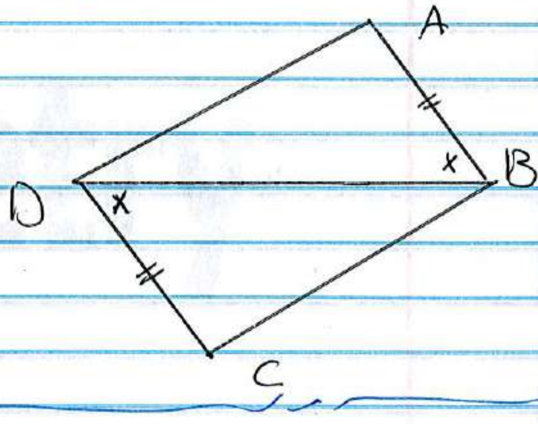
Two triangles are congruent if two sides and included angle of one triangle are congruent to the corresponding parts of the other triangle.

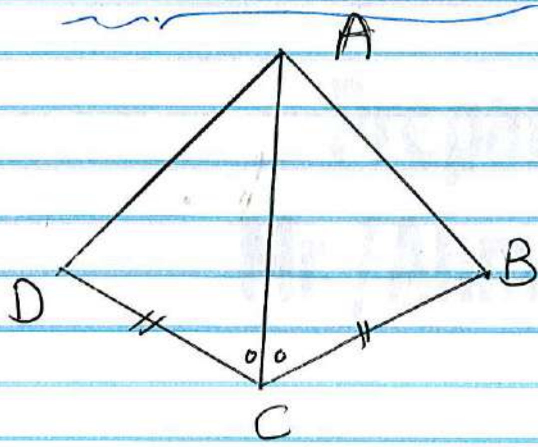
Some examples of the first case

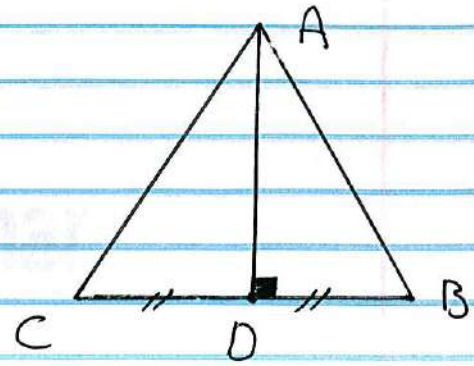








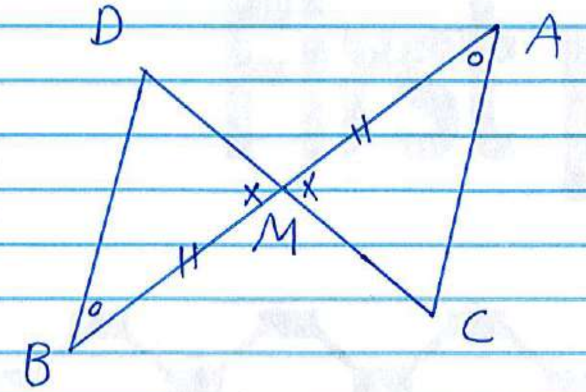


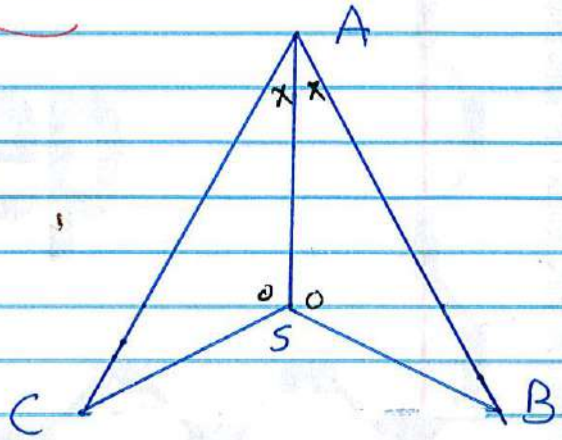


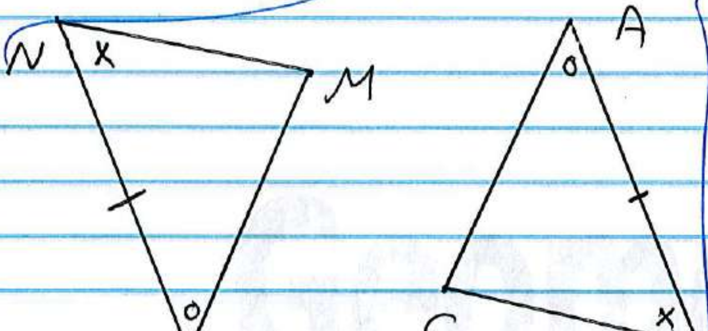
The second case of congruence of two triangles
(two angles and one side A.S.A)

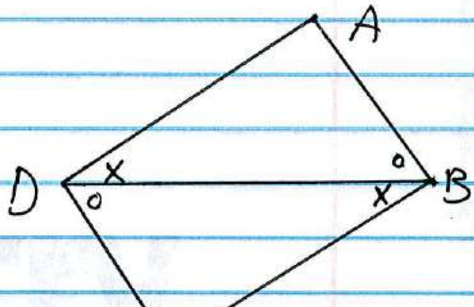
Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of other triangle.

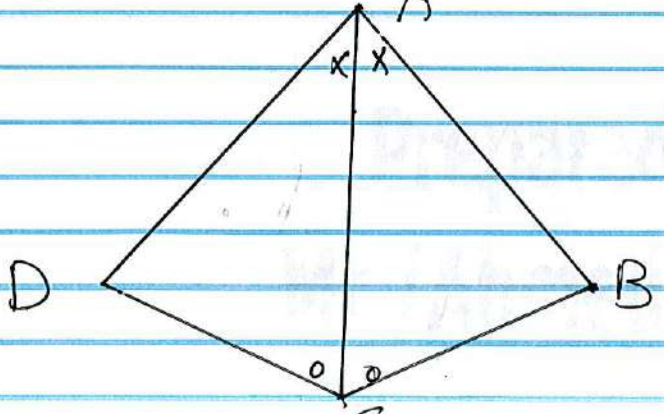
Some examples of second case:

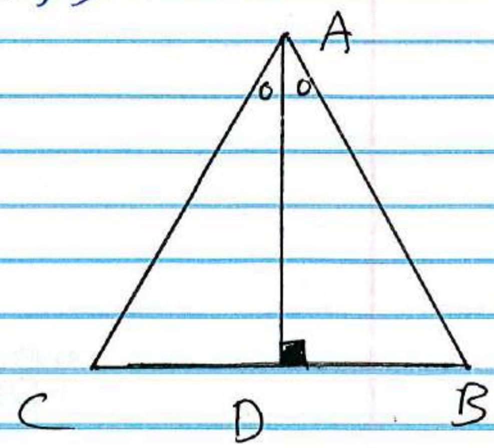








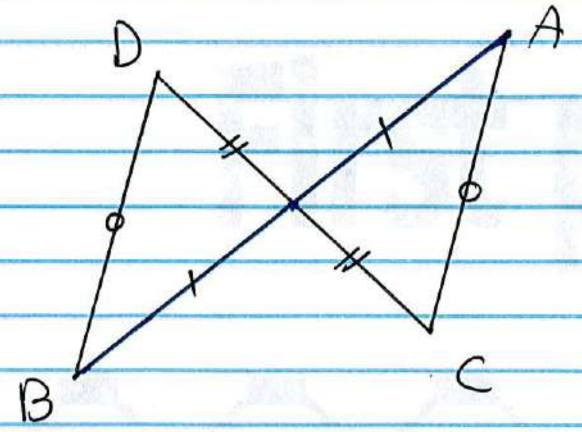


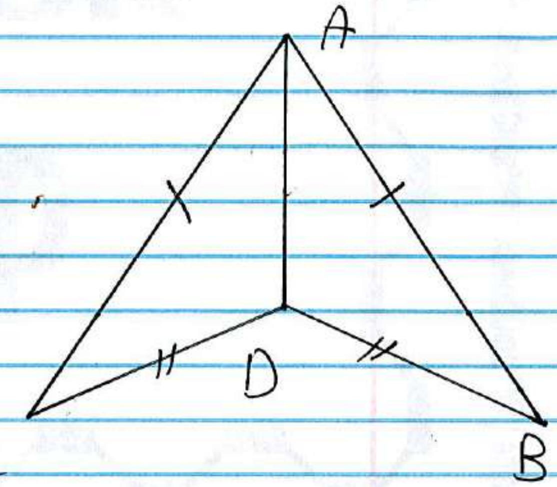


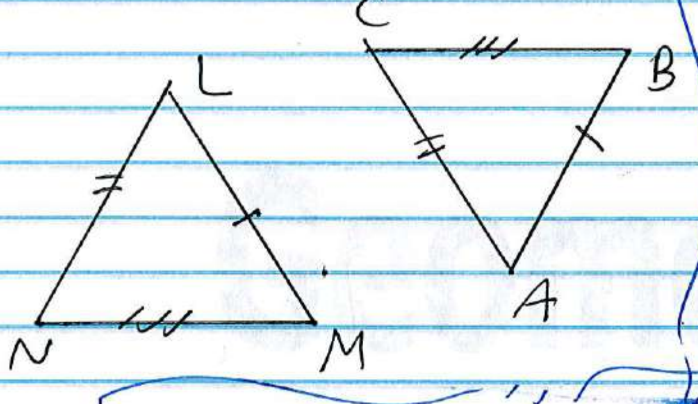
The third Case of Congruence of two triangles
(Three sides S.S.S.)

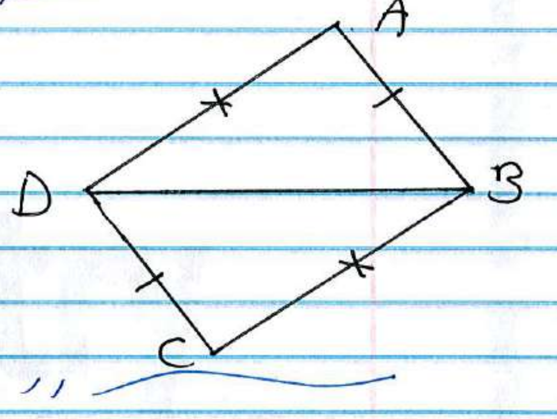
Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle

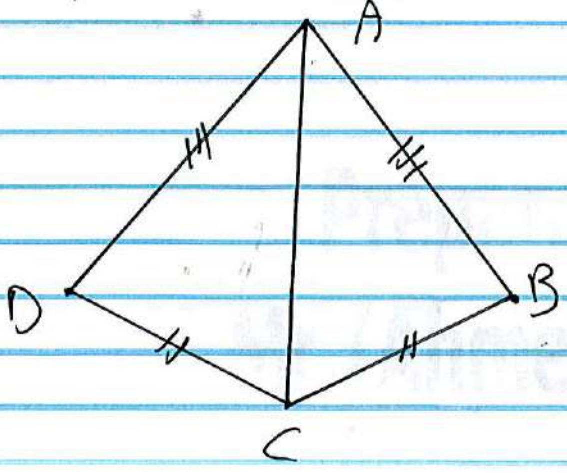
Some examples of the third Case:

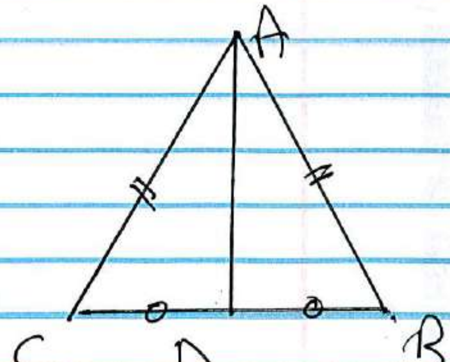












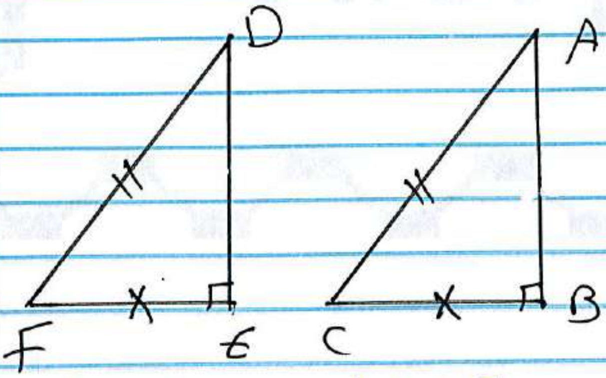
The fourth Case of Congruence of two triangle

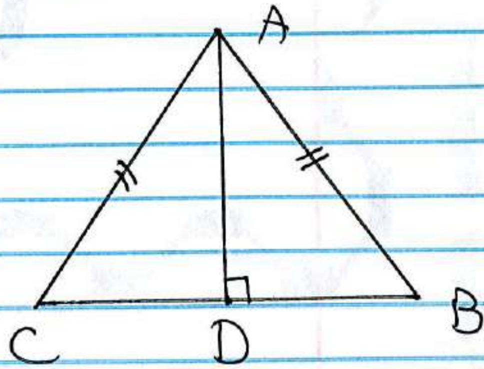
(Hypotenuse and one side in the right-angled triangle)

R.H.S

Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.

Some examples of the fourth case





If ABC and DEF are two triangle in which:

$\overline{AC} \equiv \overline{DF}$

$\overline{BC} \equiv \overline{EF}$

$m(\angle B) = m(\angle E) = 90^\circ$

, then $\Delta ABC \equiv \Delta DEF$

and we deduce that:

$\overline{AB} \equiv \overline{DE}$

$\angle A \equiv \angle D$

$\angle C \equiv \angle F$

$\overline{AB} \equiv \overline{AC}$

$\overline{AD}$ common side

$m(\angle ADB) = m(\angle ADC) = 90^\circ$

then $\Delta AB \equiv \Delta ACD$

and we deduce that

$\overline{CD} \equiv \overline{DB}$

i.e D is midpoint of $\overline{CB}$

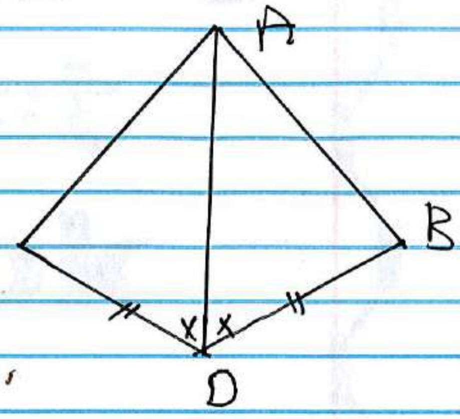
$m(\angle BAD) = m(\angle CAD)$

i.e $\overrightarrow{AD}$ bisects angle A

Ex: In the opposite figure:

$BD = DC$, $m(\angle ADB) = m(\angle ADC)$

Explain why does $\overrightarrow{AD}$ bisect angle A?



Solution

$\triangle ABD \equiv \triangle ACD$

"two sides and included angle" (S.A.S)

we deduce from the congruence that:

$m(\angle BAD) = m(\angle CAD)$

i.e: $\overrightarrow{AD}$ bisects $\angle A$

In the opposite figure:

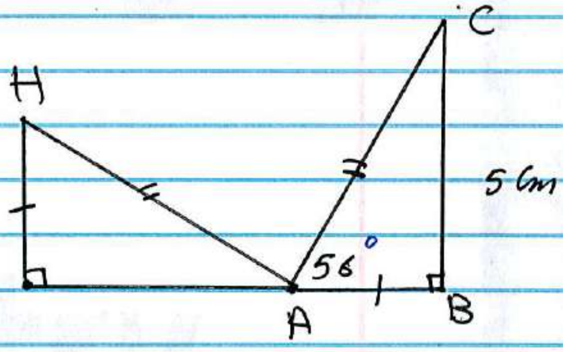
$m(\angle B) = m(\angle D) = 90^\circ$

$HD = AB$, $BC = 5\text{cm}$

$AC = AH$

Find: AD , $m(\angle AHD)$

with showing the steps of solution



Solution

$m(\angle B) = m(\angle D)$

$HD = AB$ (side)

$AC = AH$ (Hypotenuse)

then $\triangle ABC \equiv \triangle HDA$ (R.H.S)

and we deduce from the congruence that:

$AD = BC = 5\text{ cm}$

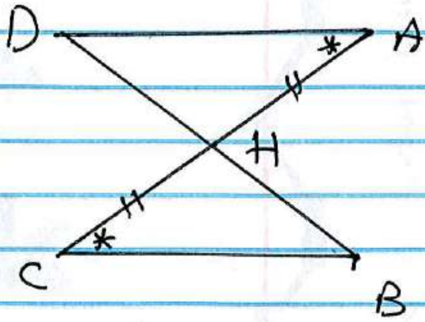
, $m(\angle AHD) = m(\angle CAB) = 56^\circ$

In the opposite figure:

$\overrightarrow{AC} \cap \overrightarrow{BD} = \{H\}$, $m(\angle A) = m(\angle C)$

H is midpoint of $\overline{AC}$

Explain why deos $\triangle AHD \equiv CHB$



Solution

$\triangle AHD \equiv \triangle CHB$

Because

$m(\angle A) = m(\angle C)$

$m(\angle AHD) = m(\angle CHB)$ (V.O.A)

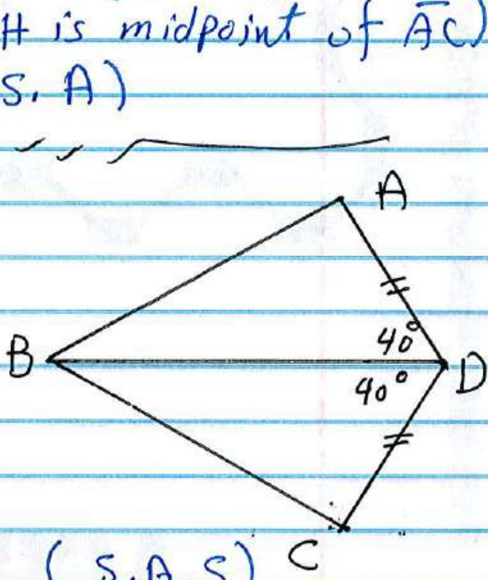
$AH = HC$ (H is midpoint of $\overline{AC}$)

Two angles and side (A.S.A)

In the opposite figure:

$AD = CD$, $m(\angle ADB) = m(\angle CDB)$

Show if the two triangles are congruent or not



Solution

$\triangle ADB \equiv \triangle CDB$

Two sides and included angle (S.A.S)

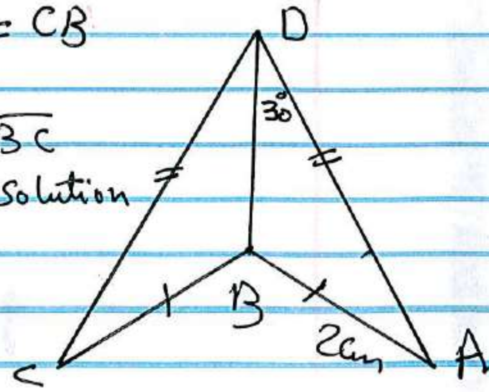
Because: $AD = DC$, $\overline{DB}$ is a common side.

$m(\angle ADB) = m(\angle CDB) = 40°$

In the opposite figure: $AD = CD$, $AB = CB$

$m(\angle ADB) = 30°$, $AB = 2cm$

Find: $m(\angle ADC)$, the length of $\overline{BC}$ with showing the steps of solution



Solution $\triangle ADB \equiv \triangle CDB$

Three sides (S.S.S)

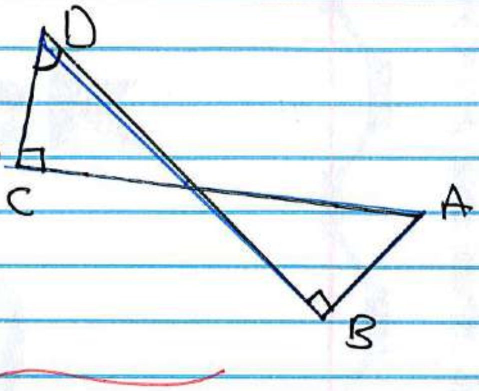
($AD = CD$, $AB = CB$, $\overline{BD}$ Common side)

we deduce from the congruence that

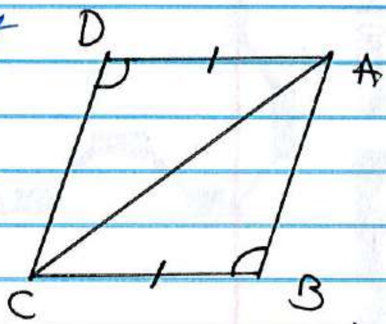
$BC = AB = 2cm$, $m(\angle ADC) = 2 \times 30 = 60°$

In each of the following figures, show if the two triangles are congruent or not
Given that the similar signs denote the congruency of the elements marked by these signs)

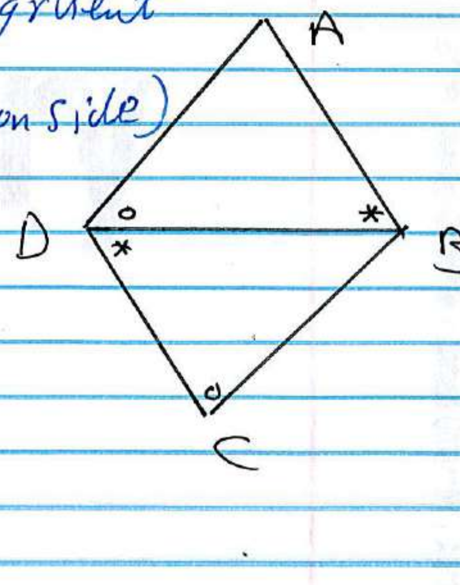
①
The two triangles are not congruent because data is not enough



The two triangles are not congruent because the given angle is not included between the two sides.



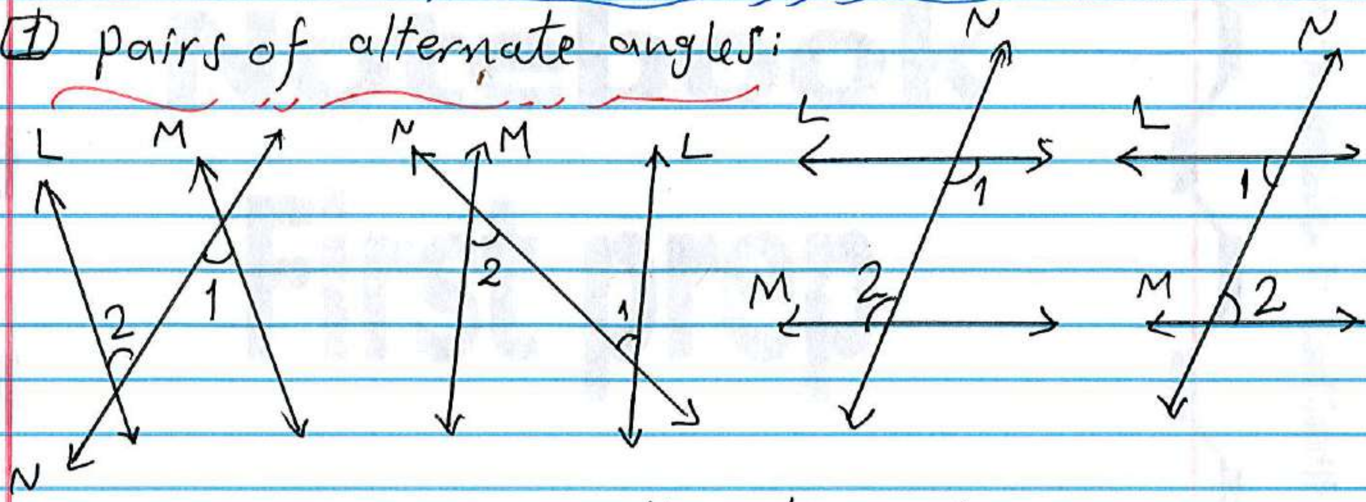
The two triangles are not congruent because the given side (Common side) is not drawn between the two angles



Parallelism

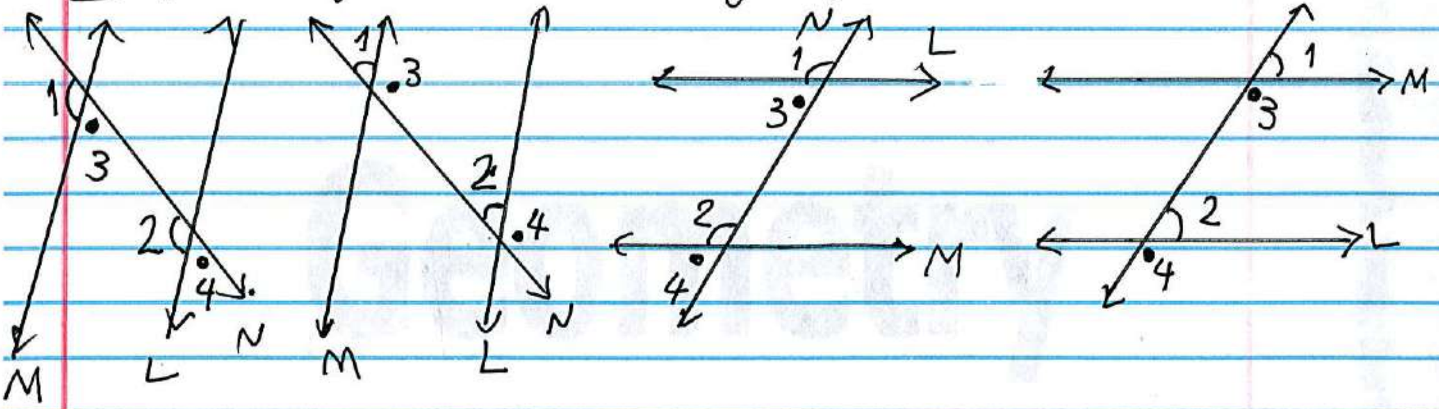
Angles formed from two straight lines and a transversal:

1 Pairs of alternate angles:



$\angle 1$ and $\angle 2$ are alternate angles.

2 Pairs of Corresponding angles:



$\angle 1$ and $\angle 2$ are Corresponding angles.
$\angle 3$ and $\angle 4$ are Corresponding angles.

3 Pairs of interior angles on the same side of the transversal:

from the previous figures.
$\angle 3$, $\angle 4$ are interior angles on the same side of the transversal.

Relation between pairs of angles formed from two parallel straight lines and a transversal to them:

If a straight line intersects two parallel straight lines, then

[1] Each two alternate angles are equal in measure.

[2] Each two corresponding angles are equal in measure

[3] Each two interior angles in the same side of the transversal are supplementary.

The Condition of parallelism of two straight lines:

The two straight lines are parallel if a third straight line intersects them and one of the following cases is satisfied:

[1] Two alternate angles have the same measure.

[2] Two corresponding angles have the same measure.

[3] Two - - angles in the same side of the transversal are supplementary.

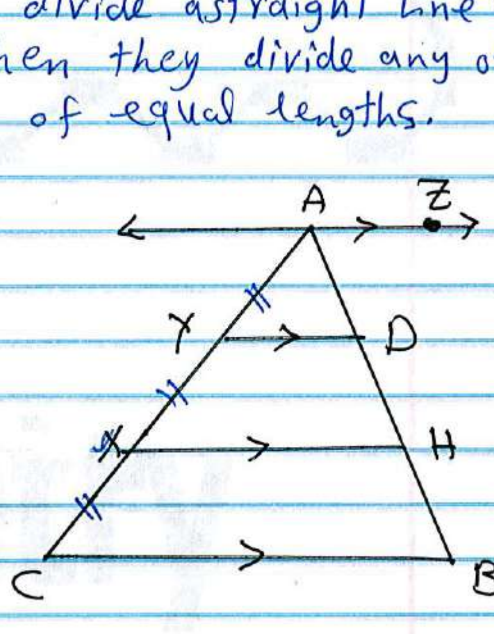
Geometric facts :-

1. The perpendicular to one of two coplaner parallel straight lines is perpendicular to the other

2. If two coplaner straight lines are perpendicular to athird one, then the two straight lines are parallel.

3. If two straight lines are parallel to athird, then these two straight lines are parallel.

4. If parallel straight lines divide astraight line into segments of equal lengths, then they divide any other straight line into segments of equal lengths.



In the opposite figure:

$\overleftrightarrow{AZ} \parallel \overleftrightarrow{YD} \parallel \overleftrightarrow{XH} \parallel CB$

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are their Transversals

, $AY = YX = XC$

Then, $AD = DH = HB$

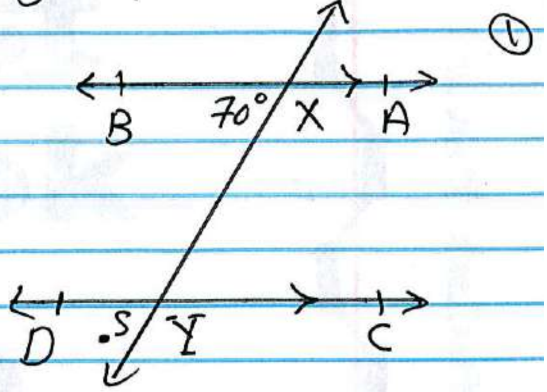
Ex: In each of the following figures, find the measure of the angle which is marked by "?"

Solution

$m(\angle DYH) = m(\angle BXY) = 70^\circ$

(alternate angles)

Because $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{XY}$ is transversal

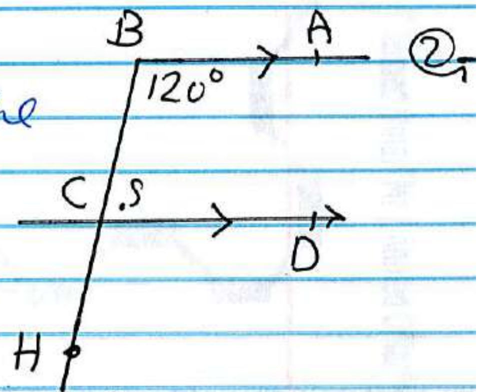


$m(\angle B) + m(\angle BCD) = 180$

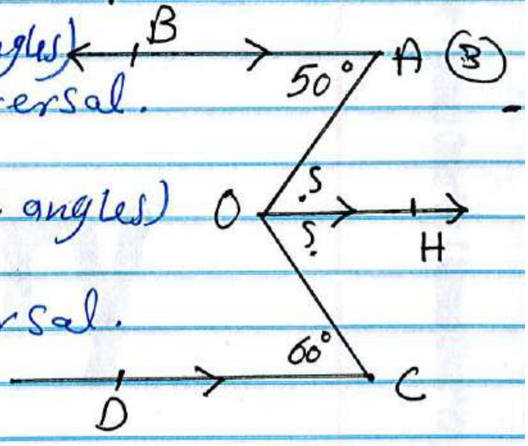
interior angles in the same side of the transversal

then:

$m(\angle BCD) = 180 - 120 = 60^\circ$

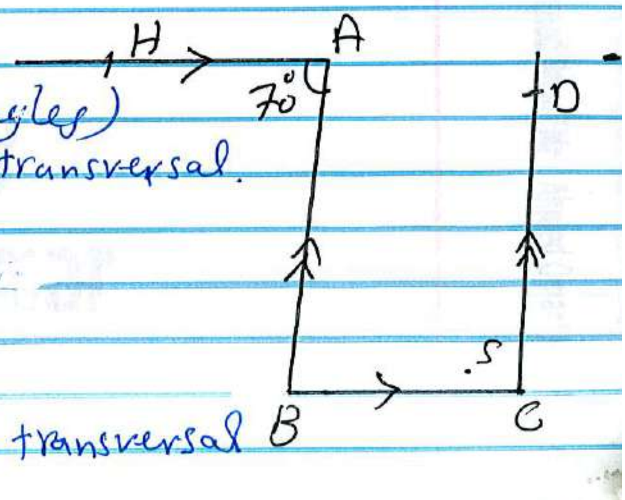


- $m(\angle AOH) = m(\angle A) = 50^\circ$ (alternate angles)
Because $\overrightarrow{AB} \parallel \overrightarrow{OH}$, $\overrightarrow{AO}$ is the transversal.
- $m(\angle HOC) = m(\angle C) = 60$ (alternate angles)
Because $\overrightarrow{OH} \parallel \overrightarrow{CD}$, $\overline{OC}$ is a transversal.



- $m(\angle B) = m(\angle A) = 70^\circ$
(alternate angles)
Because $\overrightarrow{AH} \parallel \overline{BC}$, $\overline{AB}$ is a transversal.

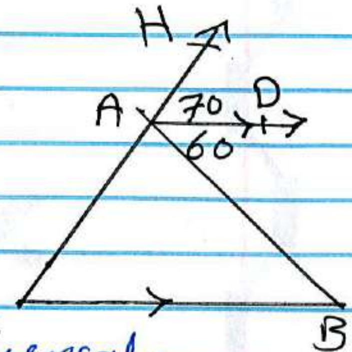
$m(\angle B) + m(\angle C) = 180^\circ$
(interior angles)
$m(\angle C) = 180^\circ - 70 = 11^\circ$
Because $\overline{AB} \parallel \overrightarrow{CD}$, $\overline{BC}$ the transversal



<u>Ex:</u> In the opposite figure

$\overrightarrow{AD} \parallel \overline{CB}$, $m(\angle DAH) = 70^\circ$

Find: $m(\angle B)$, $m(\angle C)$, $m(\angle BAC)$



<u>Solution:</u>

$m(\angle C) = m(\angle HAD) = 70^\circ$ Corresponding

Because, $\overrightarrow{AD} \parallel \overline{BC}$, $\overline{AC}$ the transversal.

$m(\angle B) = m(\angle DAB) = 60^\circ$ alternate angles.

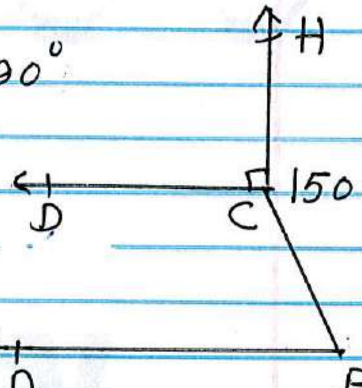
Because, $\overrightarrow{AD} \parallel \overline{BC}$, $\overline{AB}$ the transversal.

$m(\angle BAC) = 180 - (60 + 70) = 180 - 130 = 50^\circ$

In the opposite figure:

$\overrightarrow{BA} \parallel \overrightarrow{CD}$, $m(\angle BCH) = 150^\circ$, $m(\angle DCH) = 90^\circ$

Find: $m(\angle B)$.



<u>Solution:</u> $m(\angle DCB) = 360 - (90 + 150)$

$= 360 - 240$

$= 120^\circ$

Because,

The sum of measures of the accumulative angles at a point $= 360^\circ$

$m(\angle B) = 180 - 120 = 60^\circ$ (interior angles)

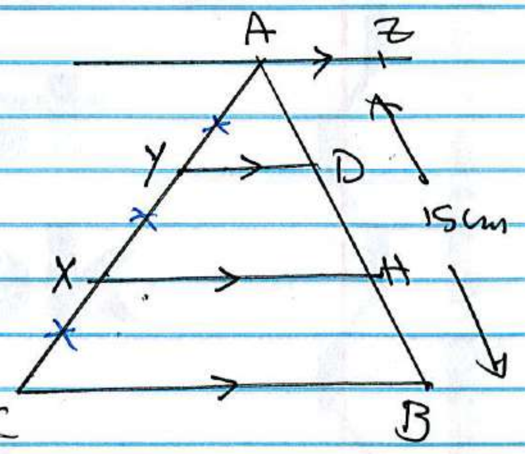
Because, $\overrightarrow{CD} \parallel \overrightarrow{BA}$, $\overline{BC}$ the transversal.

Ex. In the opposite figure:

$\overleftrightarrow{AZ} \parallel \overrightarrow{YD} \parallel \overline{XH} \parallel \overline{CB}$

, $AY = YX = XC$ AND $AB = 15$ Cm

Find the length of $\overline{BD}$.



Solution:

Since, $\overleftrightarrow{AZ} \parallel \overline{YD} \parallel \overline{XH} \parallel \overline{CB}$

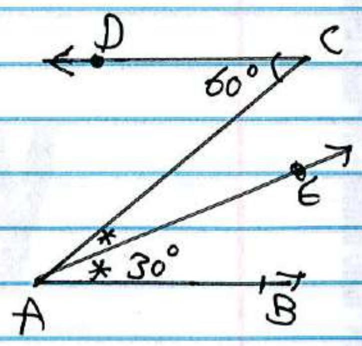
, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are their transversals, $AY = YX = XC$

then, $AD = DH = HB = \frac{15}{3} = 5$ Cm.

Then $BD = 2 \times 5 = 10$ Cm

In each of the following, Show why $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$.

Solution:



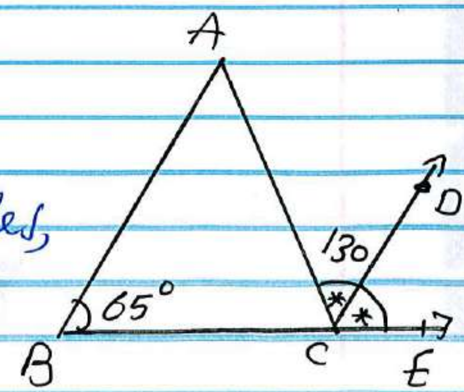
$m(\angle BAC) = 30 \times 2 = 60$

i.e. $m(\angle BAC) = m(\angle C)$

and they are two alternate angles

therefore: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$,

$m(\angle DCE) = \frac{130}{2} = 65°$

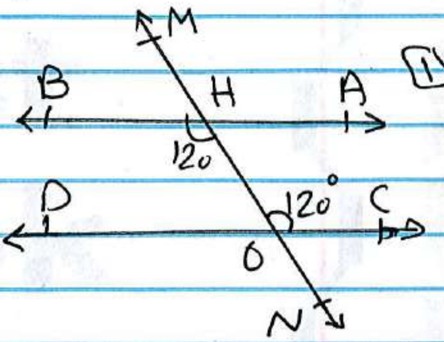


i.e. $m(\angle DCE) = m(\angle B)$

and they are corresponding angles,

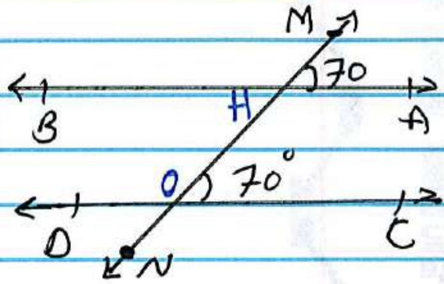
Therefore: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

In each of the following, Show why $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

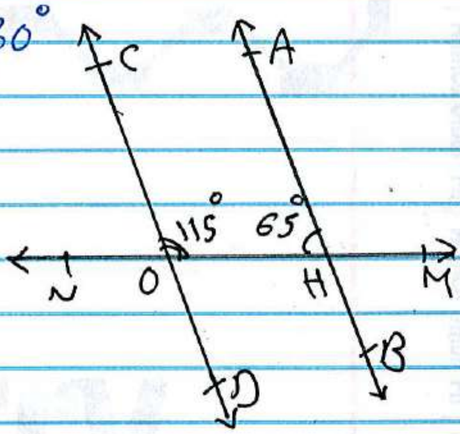


Solution $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

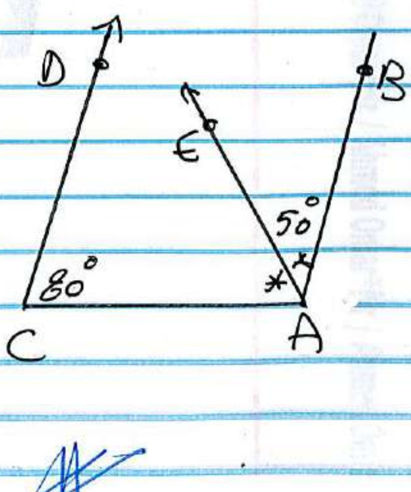
Because $m(\angle BHO) = m(\angle HOC) = 120°$
and they are alternate angles



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
Because $m(\angle AHM) = m(\angle HOC) = 70°$
and they are corresponding angles.



$m(\angle AHO) + m(\angle COH) = 115 + 65 = 180°$
and they are interior angles in one side of the transversal,
Therefore; $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



$m(\angle BAC) = 2 \times 50 = 100°$

$m(\angle BAC) + m(\angle C) = 100 + 80 = 180$

and they are interior angles in one side of the transversal.

Therefore; $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$